THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMATH5220 Complex Analysis and Its Applications 2014-2015
Assignment 1

- Due date: 28 Jan, 2015
- Remember to write down your name and student number

1. For $n \geq 1$, prove that

   (a) $1 + z + z^2 + \cdots + z^n = \dfrac{1 - z^{n+1}}{1 - z}$, if $z \neq 1$;

   (b) $1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \dfrac{1}{2} + \dfrac{\sin(n + \frac{1}{2})\theta}{2\sin\frac{\theta}{2}}$, if θ is not a multiple of 2π.

2. Let $z_1, z_2 \in \mathbb{C}^* = \mathbb{C}\backslash\{0\}$, is it true that $\mathrm{Log}(z_1 z_2) = \mathrm{Log}(z_1) + \mathrm{Log}(z_2)$? Please explain your answer.

3. Suppose $T(z) = \dfrac{az + b}{cz + d}$, with $ad - bc \neq 0$. Show that

   (a) $\lim_{z\to\infty} T(z) = \infty$ if $c = 0$;

   (b) $\lim_{z\to\infty} T(z) = \dfrac{a}{c}$ and $\lim_{z\to -d/c} T(z) = \infty$ if $c \neq 0$.

4. If $\lim_{z\to z_0} f(z) = 0$ and there exists a positive number M such that $|g(z)| \leq M$ for all z in some neighborhood of z_0, prove that $\lim_{z\to z_0} f(z)g(z) = 0$.

5. Suppose that $f(z) = \bar{z}$. By considering the Cauchy-Riemann equations, show that $f'(z)$ does not exist at any point.

6. Prove that if f and $\bar{f}$ are both analytic on a domain D, then f is constant in D.